Chaos based associative memories for pattern recognition

Martin PERNOLLET † Laurent LAVAL ††

Robotics Laboratory of Versailles (LRV)
10-12, av. de l'Europe 78140 VELIZY France

***Abstract*— *This paper deals with the interest in introducing some artificial chaotic dynamics within associative memories (namely, Attractor Neural Networks) to perform pattern recognition from either static images or images streams. In particular, a great attention is paid to the use of chaotic inhibitory interneurons, leading to propose a selective criteria which facilitates, in some sense, the discrimination of retrieved patterns. This criteria is based on the consideration of the energy of each neuron (modelized as dynamical systems) and affords to characterize the neuronal activity of each of them with respect to the probability of associated pixels (i.e. input stimuli) to belong to one or more stored pattern(s). Then, through simulation results this paper studies the effects of adding chaotic dynamics to input stimuli, from a network reactivity viewpoint. Finally, some concluding remarks and potential extensions of this work are presented.*

***Index Terms*—Associative memory, chaos, dynamical systems**

I. Introduction

In many fields of robotics related to entertainment, domestic, or medical applications ..., pattern recognition from image or video sequences plays a central role:

- in capturing some environmental properties (e.g. recognition of the ball within a soccer robots scene),
- in performing some tasks (e.g. visuomotor coordination of a robot arm),
- in enhancing human-robot interactivity (e.g. human face and gesture perception for capturing human cognition properties).

To deal with such a task (including pattern storage and retrieval, classification, finding of similar objects, ...), use of artificial neural networks has received a great attention for many years (for instance, see [4] and references therein). In particular, in the light of some investigations related to computational neuroscience, an increasing attention has been paid, over the past few years, to Attractor Neural Networks[1] (AANs). However, both design and exploitation of such networks is not trivial[2]. This motivates, for part, the growing interest in considering neuronal architectures (including neurons modelling) based on biological inspirations, as such a basis provides many pertinent and efficient actual implementations. Then, with regard to such a (biological) context, several investigations have pointed out that some human and animal brain activities exhibit chaotic (or chaotic-like) behaviors (e.g. see [2], [15], [7], [1]). In addition, some works have shown that artificial stimulation of brain by chaotic waves can someday avoid elliptic seizures, inhibit Parkinson disease, etc ... (e.g. see [3]). However, despite of these results, information processing capability related to chaotic brain dynamics is still obscure, and motivates the effort in understanding, generating and/or taming chaotic phenomena in relation with true or artificial brains. According to this purpose, the present paper investigates the contribution of chaotic signals for improving the performance of (artificial) associative memories devoted to pattern storage and recognition. In particular, the present work considers the problem of recognizing some patterns in black and white images, in case of noisy input stimuli (i.e. images with erroneous pixels which do not belong to one of the memorized patterns), and incomplete input patterns.

This paper is organized as follows. First, section II introduces some motivations and the framework of the present work. Then, section III deals with the contribution of chaotic inhibitory interneurons to pattern recognition (in relation with dynamical behavior of AAN). Then, section IV is devoted to the interest of introducing artificial chaotic dynamics within postsynaptic potential (related to excitatory neurons). Finally, some concluding remarks and potential extensions of this work are given in section V.

II. Motivations and framework

First, let us mention that, according to complexity theory, a complex system can be defined as one in which numerous independent elements continuously interact and spontaneously organize and reorganize themselves into more and more elaborate structures over time. With such a concept in mind, a natural question which arises is : Can disordered or seemingly disordered activities of elementary neurons produce some interesting, ordered, (global) behaviors of a whole AAN ?

To investigate such an idea, we consider, in this paper, a fully interconnected Attractor Neural Network (used as au-

† Master student at EFREI
†† Corresponding author: laval@lrv.uvsq.fr

[1] i.e. networks of interconnected dynamical systems as cells (i.e. neurons), for which convergence properties to particular states represent the essence of pattern recognition processes.

[2] For instance, training procedures may be CPU intensive and may produce spurious attractors and/or ill-conditioned attractor basins. Moreover, convergence of the distributed sensory cells to well defined and distinct subsets of the state space may be not guaranteed.

toassociative memory) to store and retrieve some patterns related to 16x16 pixels, black and white images (each pixel being associated with an unique excitatory neuron). Examples of two patterns to recognize are depicted in figures 1 and 2.

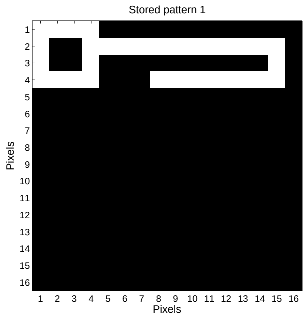

Figure 1. *Pattern 1* to retrieve: A horizontal key

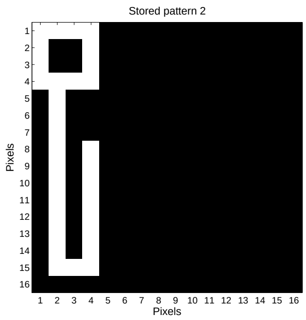

Figure 2. *Pattern 2* to retrieve: A vertical key

REMARK 1 *With regard to figures 1 and 2, let us note that the two patterns share some common pixels corresponding to a square form with a hole in the middle. Thus, in case of incomplete input patterns (i.e. stimuli) with pixels mainly relevant to this common shape, the recognition challenge is to determine which stored pattern is truly involved.*

A. The Attractor Neural Network (AAN) model

Motivated by some phenomenological observations, made by Rolls and Treves (see [17]), which suggest that CA3 region of the hippocampus behaves like an autoassociative memory (see also [10]), we consider, in this paper, a corresponding AAN as defined in [6]. This AAN consists in a fully interconnected[3] network composed of $16 \times 16 = 256$ excitatory neurons[4] plus one interneuron providing feedback inhibition (i.e. an inhibitory interneuron). During pattern learning, input stimuli cause some excitatory neurons to become active, and the synaptic connections between each pair of active neurons are then modified (Hebb Rule). Such a modification is defined here by the following relation,

$$w_{ij} = kH(R_i - 0.5M)H(R_j - 0.5M) \quad (1)$$

[3] Each excitatory neuron has 255 excitatory synaptic inputs plus one synaptic input coming from the inhibitory interneuron.

[4] Each excitatory neuron being associated with one pixel of the input image (i.e. the input stimulus) presented to the AAN.

with,

$$H(\zeta) = \begin{cases} 1 & \zeta > 0 \\ 0 & \zeta \leq 0 \end{cases} \quad (2)$$

where $M \in \mathbb{R}^{+*}$ is the maximum firing rate of the neurons (here, $M = 100$ spikes/s), $w_{ij} \in \mathbb{R}^{+}$ is the (tunable) weight of the synaptic connection between the i^{th} excitatory neuron and the j^{th} one. R_i (resp. R_j) is the firing rate of the i^{th} (resp. j^{th}) neuron. $k \in \mathbb{R}^{+*}$ is a weighting factor (here, $k = 0.016$)

REMARK 2

- *According to (1) and (2), synaptic connexions are strengthened only when both i^{th} and j^{th} involved neurons are firing at greater than half their maximum (firing) rates. In this case, the synaptic weight w_{ij} changes to the fixed value k.*
- *As pairs of neurons are reciprocally interconnected, this implies that synaptic connections are modified identically (i.e. $w_{ij} = w_{ji}$)*

Then, from a mathematical viewpoint, the excitatory and inhibitory parts of the AAN can be defined respectively as follows.

Excitatory part:

Based on spike rate descriptions, dynamical behavior of each excitatory neuron i can be defined by (see [6]),

$$10\frac{dR_i}{dt} = -R_i + \frac{\sigma^2 \times PSP_i^2}{\sigma^2 + PSP_i^2}, \quad R_i(0) = 0 \quad (3)$$

where $R_i \in \mathbb{R}^{+}$ is the firing rate of the i^{th} excitatory neuron ($i = 1...256$), $\sigma \in \mathbb{R}^{+*}$ is the semi-saturation constant (here, $\sigma = 10$), and PSP_i characterizes the postsynaptic potential defined by

$$PSP_i = S_i + \sum_{j=1}^{256}(w_{ij} \times R_j) - \delta \times G \quad (4)$$

where $G \in \mathbb{R}^{+}$ is the firing rate of the inhibitory interneuron, $\delta \in \mathbb{R}^{+*}$ is a weighting factor (here, $\delta = 0.1$), w_{ij} is defined by Eq. (1), and S_i characterizes the stimulus exciting the neuron i during a finite time $t_{stimulation}$. As images presented to the network are black and white ones, S_i can be defined as,

$$\begin{array}{l} if \quad t \leq t_{stimulation} \begin{cases} S_i = 0 & \leftarrow \quad black\,pixel \\ S_i = 1 & \leftarrow \quad white\,pixel \end{cases} \\ else \quad S_i = 0 \end{array} \quad (5)$$

Inhibitory part: In order to perform a comparative study, we consider either non-chaotic or chaotic inhibitory interneurons defined respectively as follows.

Non-chaotic inhibitory interneuron:

$$10\frac{dG}{dt} = -G + \gamma_G \sum_{i=1}^{256} R_i, \quad G(0) = 0 \quad (6)$$

where $\gamma_G \in \mathbb{R}^{+*}$ is an input synaptic weight (assumed to be here: $\gamma_G = 0.076$), $G \in \mathbb{R}^{+}$ is the firing rate of the inhibitory interneuron.

Chaotic inhibitory interneuron:

By considering the objective of the present work and the common use of (equivalent) electrical circuits as neuron representation (e.g. [14], [8]), we propose to consider here an electronic implementation of a chaotic circuit, referred to as Chua's circuit, as inhibitory interneuron with chaotic dynamics (see Fig. 4). This chaotic oscillator can be formulated as a dimensionless system of the form (see [16]),

$$\dot{X} = f(X) \;\Leftrightarrow\; \begin{cases} \dot{x} = \alpha\,(y - x - f_d(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y \end{cases} \tag{7}$$

where $X = [x\,y\,z]^T$ is the state vector. α, β and γ are system parameters. The nonlinear diode of Chua is characterized by: $f_d(x) = bx + \frac{1}{2}(a-b)\,(|x+1|\,|x-1|)$ where a and b are two constant parameters. Here, $\alpha = 10$, $\beta = 14.87$, $\gamma = 1$, $a = -1.27$, and $b = -0.68$.

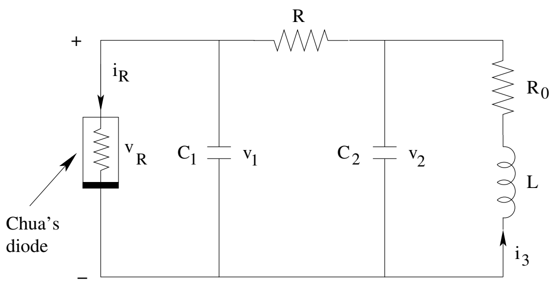

Figure 3. Basic electrical circuit of Chua's oscillator

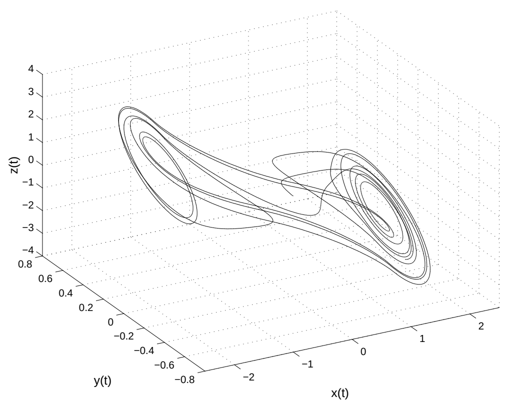

Figure 4. Example of Chua's oscillator chaotic trajectory

We then propose to consider a chaotic inhibitory interneuron defined by,

$$G = E\,\gamma_G \sum_{j=1}^{256} R_j \tag{8}$$

with

$$E = \frac{1}{2}\left(x^2 + \alpha y^2 + \frac{\alpha}{\beta} z^2\right) \tag{9}$$

where E is a positive definite function which characterizes the energy of the Chua's circuit, and $\gamma_G \in \mathbb{R}^{+*}$ is a weighting factor.

Remarks 3

- *According to (8)-(9), firing rate G of the chaotic inhibitory cell is always positive-valued for all time t (with some analogy to the non chaotic one defined by Eq. (6)).*
- *All excitatory neurons are connected to the inhibitory interneuron which provides recurrent subtractive inhibition (see Eqs. (3) and (4)) to regulate the neuronal activity (i.e. to lead to a balance between inhibition and excitation). Thus, magnitude of G, and therefore tuning of γ_G (within an admissible range), is of a crucial importance to obtain either convergence of the AAN towards stable steady states (case of AAN with non-chaotic inhibitory cell) or specific topological configurations with invariant properties (case of AAN with a chaotic inhibitory interneuron, as discussed further in section III.B). Here, we will consider:* $0.025 < \gamma_G < 0.75$ *in case of non-chaotic inhibitory neuron, and a smaller range* $0.0226 < \gamma_G < 0.0229$ *in case of a chaotic one.*
- *According to (8) and (9), the chaotic inhibitory interneuron behaves like a dynamical system, as implicitly related, through the term E, to ordinary differential equations characterizing the Chua's circuit.*

III. Contribution of a chaotic inhibitory interneuron

Before to deal with the contribution of a chaotic inhibitory interneuron to the whole AAN dynamical behavior, this section first points out some practical conditions inherent to the use of such a kind of interneuron.

A. Role of initial conditions (of the chaotic circuit)

It is well-known that balance between inhibition and excitation plays a crucial role in generating of neuronal activity and, therefore, in leading to successful recognition of patterns. Thus, according to Eqs. (8) and (9), success and even performance of pattern recognition clearly depends on the magnitude of E and, therefore, on the orbit the Chua's circuit is tracking during the corresponding temporal window. Therefore, according to properties of chaotic systems[5], initial conditions of Chua's circuit have to be carefully selected (excluding random initial conditions to guaranty the repeatability of the recognition processes)[6]. For instance, Figs. 6 and 7 show respectively unsuccessful and successful recognition of a stored pattern, in case of same input stimuli depicted in Fig. 5, same stimulation time, and two different but very close sets of initial conditions. Indeed Fig. 6 shows that neuronal activity of each excitatory neuron converges towards zero (meaning that pattern recognition failed). Conversely, with small changes

[5] in case of deterministic chaos

[6] random initial conditions have also to be avoided to guaranty the chaotic motion of Chua's circuit

of initial conditions ($x(0) = 0.320$ instead of $x(0) = 0.350$), the pattern recognition succeeds as shown through Fig. 7.

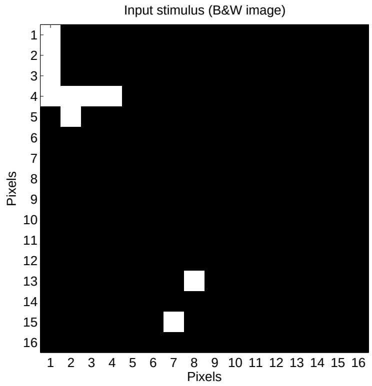

Figure 5. Input stimulus (incomplete vertical key with erroneous pixel as additive noise)

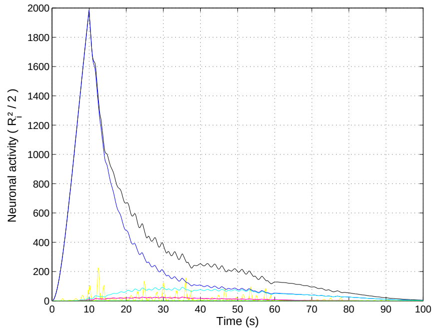

Figure 6. Neuronal activity of some representative neurons in case of initial conditions: $x(0) = 0.350$, $y(0) = 0.155$, $z(0) = -0.008$ (see Appendix I for correspondence between colored plots and involved neurons)

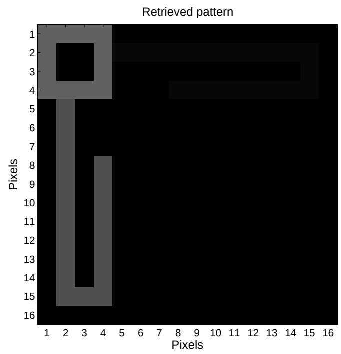

Figure 7. Recognized pattern in case of initial conditions: $x(0) = 0.320$, $y(0) = 0.155$, $z(0) = -0.008$)

B. *Contribution of a chaotic inhibitory interneuron*

In order to evaluate the interest of introducing chaotic dynamics within the inhibitory interneuron behavior, let us first consider some simulations results coming from the consideration of the *non-chaotic* AAN (as defined by Eqs. (3) and (6)). Moreover, let us consider the input stimuli of Fig. 5, whom particularity is that only one pixel can lead to discriminate between patterns *1* and *2* (see Figs. 1 and 2). Then, nearly optimal tuning of parameter γ_G (that is $\gamma_G = 0.0732$) leads to the neuronal activity depicted in figure 8 (corresponding to a successful retrieval of pattern 1).

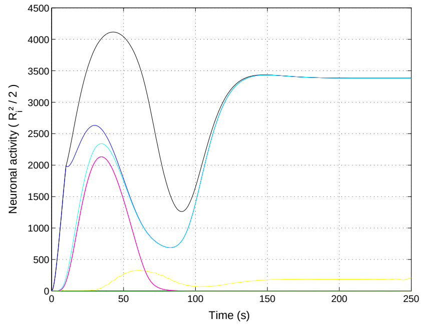

Figure 8. Neuronal activity of some representative neurons in case of non chaotic AAN (see Appendix I for correspondence between colored plots and involved neurons)

Figure 8 clearly shows that the firing rate R_i of each neuron can take only two distinct values once the AAN has reached a stable state. Thus, with such a non-chaotic AAN, pattern recognition is of a binary type (a stored pattern, but not necessary the true one, is retrieved or not). Now repeating the same experiment[7] with the consideration of a chaotic inhibitory interneuron (see Eq. (8)), we then obtain some representative neurons behaviors as shown in figure 9 (which also corresponds to retrieval of the true pattern.)

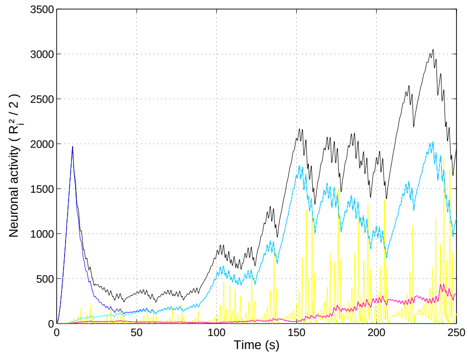

Figure 9. Neuronal activity of some representative neurons in case of AAN with chaotic inhibitory interneuron (see Appendix I for correspondence between colored plots and involved neurons)

With regard to figure 9, we can remark that:

- Firing rate of neurons can take several distinct values.
- Firing rates of all excitatory neurons which do not belong to one of the memorized patterns, stay or converge towards zero (depending on both initial stimulation (case of an erroneous pixel) and coupling evolution during the recognition process).
- Firing rates of excitatory neurons which belong to one of the memorized pattern, do not converge towards steady state values, meaning that the AAN does not reach any stable state. However, we can note that the AAN reaches a topological configuration with invariant properties. Indeed, after a short time t (for instance $t = 50s$ in case of figure 9), a relative scaling of firing rates appears. With some analogy with

[7] with same stimulation time

fuzzy logic, this scaling follows a rule of the form : *The highest is the magnitude of the firing rate the highest is the probability of the (excited) neuron to belong to the true pattern to retrieve.*

As illustration of this scaling rule, Fig. 9 shows that neurons associated with the form shared by the two memorized patterns own the highest firing rate magnitude (black plot in fig. 9), as these neurons must belong to the right pattern. Conversely, neurons associated with the horizontal key shape own the lowest non-zero firing rate. By using a greyscale coding to characterize the neuronal activity we then obtain the following figure 10.

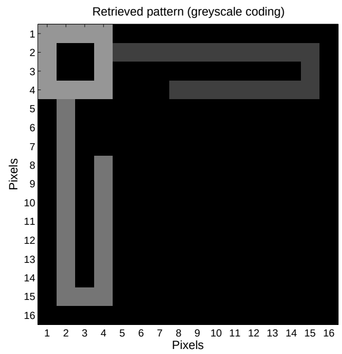

Figure 10. Greyscale coding of neuronal activity in case of AAN with chaotic inhibitor

Consequently, introducing of chaotic dynamics within the inhibitory cell can facilitate, in some sense, the discrimination between retrieved patterns (in particular when dealing with highly incomplete stimuli).

Remark 4 *In case of non-chaotic AAN, reach of a vicinity of the steady-state has to be effective before to conclude on the retrieved pattern. In case of AAN with chaotic inhibitory interneuron, some conclusions can be made once the relative scaling of firing rates is effective. Then, when comparing results of figures 8 and 9, chaotic ANN appears to be faster, and then more suitable to envisage the treatment of images streams.*

With regard to figure 9, such a plot is not really suitable for quantitative discrimination as it involves to define a magnitude threshold to distinguish between neurons which really belong to the pattern and those which may or do not belong to the right pattern (even if, in the last case, the firing rate is close to zero). To overcome this threshold design, we propose to make use of an another criteria that is to characterize the activity of each excitatory neurons versus the inhibitory one. As a result, we then obtain a plot as in figure 11. According to that, discrimination can be made by means of considering the areas where the neuronal activity belongs, in the mean. More precisely, by considering the bisectrix (which characterizes a magnitude of the excitatory neuron equal to those of the inhibitory one), averaged neuronal activity lying up the bisectrix corresponds to a neuron which belongs to the right pattern to retrieve (conversely an averaged firing rate lying on or down the bisectrix corresponds to a neuron which actually do not belong to the pattern). Moreover, with regard to

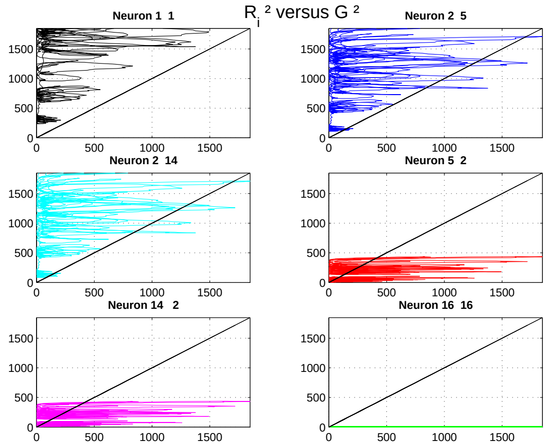

Figure 11. Neuronal activity of the chaotic AAN (see Appendix I for correspondence between colored plots and involved neurons)

the upper part, the distance between the activity area and the bisectrix characterizes the probability that a neuron belongs to the pattern.

IV. Chaotic excitation

As pointed out in [5], introducing of a noisy signal (namely, white noise) in addition with the stimuli, can enhance the reactivity of a neural network. Such a reactivity is of great importance in order to deal with streams of images rather than static images. Thus, this section is devoted to evaluate the interest in introducing artificial chaotic signals (instead of white noise) within the stimulus of each excitatory neurons. In this objective, we consider here a postsynaptic potential (PSP) of the form,

$$PSP_i = S_i + \left(\sum_{j=1}^{256} w_{ij} R_{ij} \right) - 0.1 \times G + S_E \qquad (10)$$

where firing rate G is coming from a *non-chaotic* inhibitory neuron (as defined by Eq. (6)) and S_E is an additive, artificial, chaotic signal that we propose to be of the form,

$$S_E = \frac{1}{2} \times E \times \gamma_E \times \Sigma R_j \qquad (11)$$

where E is given by Eq. (9)

Remark 5 *Parameter γ_E is a weighting factor to tune carefully in order to keep on some dynamical properties of the network (i.e. the effects of image stimuli) while maintaining an artificial, remaining excitatory signal.*

First, let us consider a *non-chaotic* AAN stimulated by an image as depicted in figure 12. In this case, the image must be presented during at least $t_{stimulation} = 1.44s$ to obtain some effective neuronal activities (leading to the true pattern retrieval), as shown in figure 13.

Now, in order to make some comparisons, let us consider the AAN with “chaotic” postsynaptic potentials (see Eq. 10). Then, appropriate choice of initial conditions for the Chua’s circuit and tuning of parameter γ_E (here $\gamma_E = 0.28$), enable to limit to a stimulation time $t_{stimulation} \leq 0.6s$, as shown through results of figure 14.

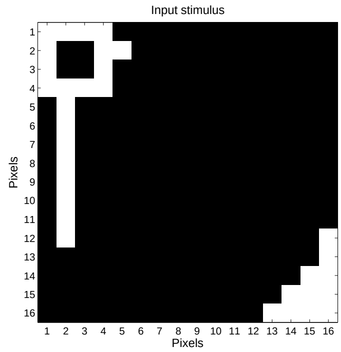

Figure 12. Incomplete and noisy stimulus

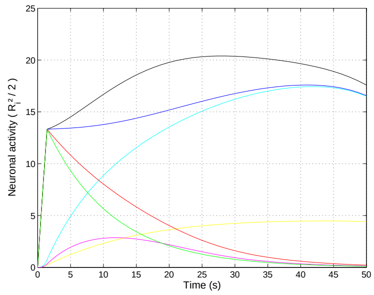

Figure 13. Neuronal activity of the non chaotic AAN in case of stimulation time $t_{stimulation} = 1.44$ (see Appendix I for correspondence between colored plots and involved neurons)

Therefore, addition of remaining chaotic signals to stimulation ones enhance the global reactivity of the network. However, a small stimulation time leads to a huge duration before the network reaches a steady state. Moreover, such an additive signal excites the whole set of excitatory neurons (including those which do not belong to any stored pattern). This leads, in fine, to artificially increase the firing rate of neurons associated to a wrong pattern (see figure 15 based on greyscale coding), and to intricate the discrimination.

V. Conclusion

This paper has shown that introducing of artificial chaotic dynamics within inhibitory interneurons leads the whole ANN behavior to exhibit topological configurations with invariant properties, instead of a finite number of steady states. When dealing with pattern recognition, such an emergence of configurations facilitates, in some sense, the discrimination of retrieved pattern, with some analogy with fuzzy logic. This paper has also shown that remaining artificial chaotic signal in addition with input stimuli, enhances the reactivity of the network, and affords to limit the stimulation time (for pattern recognition). However, such a signal excites the whole set of neurons, and intricate the discrimination of retrieved patterns. Future works will deal with coupling of neurons in order to address the problem of pattern recognition with AAN in case of geometrical transformations of input patterns (dilatation, rotations ...).

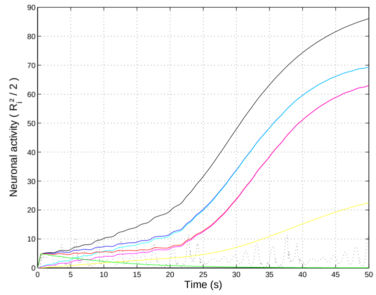

Figure 14. Neuronal activity of the chaotic AAN in case of $t_{stimulation} = 0.5s$ (see Appendix I for correspondence between colored plots and involved neurons)

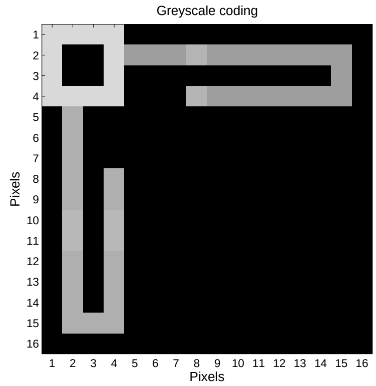

Figure 15. Greyscale coding of neuronal activity in case of $t_{stimulation} = 0.5s$

References

[1] M.I. Rabinovich and H.D.I. Abarbanel, “The role of chaos in neural systems”, Neu roscience, Vol. 87, No 1, pp. 5-14, 1998.

[2] A. Babloyantz, C. Nicolis, and J.M. Salazar, “Evidence of chaotic dynamics of brain activity during the sleep cycle”, Phys. Lett., Vol. 111 A, pp. 152-156, 1985.

[3] A. Babloyantz and A. Destexhe, “Low-dimensional chaos in instance of epilepsy”, Proc. Nat. Acad. Sci., USA, pp. 3513-3517, 1986.

[4] C.M. Bishop, “Neural Networks for Pattern Recognition”, Oxford University Press, 1995.

[5] C.M. Bishop, ”Training with Noise is Equivalent to Tikhonov Regularization”, Neural Computation, Vo. 7, No.1, pp. 108-116, 1995.

[6] H.R. Wilson, “Spikes, decisions, and actions: The dynamical foundations of neuroscience”, Oxford University Press, 1999.

[7] L. Glass “Chaos in neural systems. The Handbook of Brain Theory and Neural Networks”, pp. 186?189. MIT Press, 1995.

[8] D. Johnston and S.M-S. Wu, “Foundations of Cellular Neurophysiology”, MIT Press, Third ed., 1997.

[9] T. Kohonen. Self-Organizing Maps. Information Sciences. Springer, second edition, 1997.

[10] D. Marr, “A theory for archicortex”, Phil. Trans. R. Soc. Lond., B 262, pp. 23-81, 1971.

[11] I. Parberry, “A primer on the complexity theory of neural networks”, In Formal Techniques in Artificial Intelligence: A Source book, pp. 217-268, Elsevier, 1990

[12] O. Hendin, D. Horn, and M.V. Tsodyks, “The role of inhibition in an associative memory model of the olfactory bulb”, Journal of Computational Neuroscience, Vol. 4, pp. 173-182, 1997.

[13] R.N. Madam, “Chua’s circuit : A paradigm for chaos”, World Scientific Series on Nonlinear Science. Series B, Special Theme Issues and Proceedings Vol. 1 (World Scientific, Singapore), 1993.

[14] G.M. Shepherd, “Neurobiology”, Oxford Univ.Press, 1994.

[15] C.A. Skarda, W.J. Freeman, “How brain makes chaos in order to make sense of the world”, Behav. Brain Sci., Vol. 10, pp. 161-195, 1987.

[16] T. Yang and L.O. Chua, “Control of chaos using sampled-data feedback control”, Int. Journal of Bifurcation and Chaos, Vol. 8, pp. 2433-2438, 1998.

[17] E.T. Rolls and A. Treves, “Neural networks and brain function”, Oxford University Press, 1998.

Appendix

A. Appendix I

Preliminary remark: For all concerned figures, neuronal activity is characterized by the quantity $\frac{1}{2}R_i^2$ (where R_i is the firing rate of the i^{th} neuron). This quantity is assumed to represent the energy of each neuron (modelized as dynamical system).

Correspondence between colored plots and neurons

Deep blue	Excited neuron belonging to the true pattern to retrieve
Light blue	Non-excited neuron belonging to the true pattern to retrieve
Deep red	Excited neuron which does not belong to the true pattern to retrieve
Light red	Non-excited neuron which does not belong to the true pattern to retrieve
Black	Excited neuron belonging to the two patterns
Yellow	Inhibitory neuron
Green	Excited neuron with an erroneous pixel (noisy pixel)